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Homomorphisms between Poisson JC^*-Algebras

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Abstract. It is shown that every almost linear mapping $h : \mathcal{A} \to \mathcal{B}$ of a unital Poisson JC^*-algebra $\mathcal{A}$ to a unital Poisson JC^*-algebra $\mathcal{B}$ is a Poisson JC^*-algebra homomorphism when $h(2^n u \circ y) = h(2^n u) \circ h(y)$, $h(3^n u \circ y) = h(3^n u) \circ h(y)$ or $h(q^n u \circ y) = h(q^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all unitary elements $u \in \mathcal{A}$ and $n = 0, 1, 2, \cdots$, and that every almost linear almost multiplicative mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism when $h(2x) = 2h(x)$, $h(3x) = 3h(x)$ or $h(qx) = qh(x)$ for all $x \in \mathcal{A}$. Here the numbers $2, 3, q$ depend on the functional equations given in the almost linear mappings or in the almost linear almost multiplicative mappings.

Moreover, we prove the Cauchy–Rassias stability of Poisson JC^*-algebra homomorphisms in Poisson JC^*-algebras.

Keywords: Poisson JC^*-algebra homomorphism, Poisson JC^*-algebra, stability, linear functional equation.

Mathematical subject classification: Primary: 46L05, 39B52; Secondary: 47B48, 17B63.

1 Introduction

A *Poisson C^*-algebra* $\mathcal{A}$ is a C^*-algebra with a $\mathbb{C}$-bilinear map $\{\cdot, \cdot\} : \mathcal{A} \times \mathcal{A} \to \mathcal{A}$, called a *Poisson bracket*, such that $(\mathcal{A}, \{\cdot, \cdot\})$ is a complex Lie algebra and

$$\{ab, c\} = a\{b, c\} + \{a, c\}b$$

for all $a, b, c \in \mathcal{A}$. Poisson algebras have played an important role in many mathematical areas and have been studied to find sympletic leaves of the corresponding Poisson varieties. It is also important to find or construct a Poisson bracket in the theory of Poisson algebra (see [3, 7, 8, 20]).

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The original motivation to introduce the class of nonassociative algebras known as Jordan algebras came from quantum mechanics (see [19]). Let $\mathcal{L}(\mathcal{H})$ be the real vector space of all bounded self-adjoint linear operators on $\mathcal{H}$, interpreted as the (bounded) *observables* of the system. In 1932, Jordan observed that $\mathcal{L}(\mathcal{H})$ is a (nonassociative) algebra via the *anticommutator product* $x \circ y := \frac{xy+yx}{2}$. A commutative algebra X with product $x \circ y$ is called a *Jordan algebra*. A unital Jordan C^*-subalgebra of a C^*-algebra, endowed with the anticommutator product, is called a *JC^*-algebra*.

Let X and Y be Banach spaces with norms $||\cdot||$ and $\|\cdot\|$, respectively. Consider $f: X \to Y$ to be a mapping such that $f(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in X$. Assume that there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that

$$\|f(x+y) - f(x) - f(y)\| \leq \theta(||x||^p + ||y||^p)$$

for all $x, y \in X$. Rassias [12] showed that there exists a unique $\mathbb{R}$-linear mapping $T : X \to Y$ such that

$$\|f(x) - T(x)\| \leq \frac{2\theta}{2 - 2^p}||x||^p$$

for all $x \in X$. Găvruta [2] generalized the Rassias' result: Let G be an abelian group and Y a Banach space. Denote by $\varphi: G \times G \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x, y) = \sum_{j=0}^{\infty} 2^{-j}\varphi(2^j x, 2^j y) < \infty$$

for all $x, y \in G$. Suppose that $f : G \to Y$ is a mapping satisfying

$$\|f(x+y) - f(x) - f(y)\| \leq \varphi(x, y)$$

for all $x, y \in G$. Then there exists a unique additive mapping $T : G \to Y$ such that

$$\|f(x) - T(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x)$$

for all $x \in G$. C. Park [9] applied the Găvruta's result to linear functional equations in Banach modules over a C^*-algebra.

Jun and Lee [4] proved the following: Denote by $\varphi: X\backslash\{0\} \times X\backslash\{0\} \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x, y) = \sum_{j=0}^{\infty} 3^{-j}\varphi(3^j x, 3^j y) < \infty$$

for all $x, y \in X \setminus \{0\}$. Suppose that $f : X \to Y$ is a mapping satisfying

$$\|2f\big(\frac{x+y}{2}\big) - f(x) - f(y)\| \le \varphi(x, y)$$

for all $x, y \in X \setminus \{0\}$. Then there exists a unique additive mapping $T : X \to Y$ such that

$$\|f(x) - f(0) - T(x)| \le \frac{1}{3}\big(\widetilde{\varphi}(x, -x) + \widetilde{\varphi}(-x, 3x)\big)$$

for all $x \in X \setminus \{0\}$. C. Park and W. Park [11] applied the Jun and Lee's result to the Jensen's equation in Banach modules over a C^*-algebra.

Recently, Trif [18] proved the following: Let $q := \frac{l(d-1)}{d-l}$, $r := -\frac{l}{d-l}$. Denote by $\varphi : X^d \to [0, \infty)$ a function such that

$$\widetilde{\varphi}(x_1, \cdots, x_d) = \sum_{j=0}^{\infty} q^{-j}\varphi(q^j x_1, \cdots, q^j x_d) < \infty$$

for all $x_1, \cdots, x_d \in X$. Suppose that $f : X \to Y$ is a mapping satisfying

$$\|d\ {}_{d-2}C_{l-2}\, f\left(\frac{x_1 + \cdots + x_d}{d}\right) + {}_{d-2}C_{l-1} \sum_{j=1}^{d} f(x_j)$$
$$-l \sum_{1\le j_1<\cdots<j_l\le d} f\left(\frac{x_{j_1} + \cdots + x_{j_l}}{l}\right)\| \le \varphi(x_1, \cdots, x_d)$$

for all $x_1, \cdots, x_d \in X$. Then there exists a unique additive mapping $T : X \to Y$ such that

$$\|f(x) - f(0) - T(x)\| \le \frac{1}{l \cdot {}_{d-1}C_{l-1}}\widetilde{\varphi}(qx, \underbrace{rx, \cdots, rx}_{d-1 \text{ times}})$$

for all $x \in X$. And C. Park [10] applied the Trif's result to the Trif functional equation in Banach modules over a C^*-algebra. Several authors have investigated functional equations (see [1], [13]–[17]).

Throughout this paper, let $q = \frac{l(d-1)}{d-l}$ and $r = -\frac{l}{d-l}$ for positive integers l, d with $2 \le l \le d-1$. Let $\mathcal{A}$ be a unital Poisson JC^*-algebra with norm $||\cdot||$, unit e and unitary group $\mathcal{U}(\mathcal{A})$, and $\mathcal{B}$ a unital Poisson JC^*-algebra with norm $\|\cdot\|$ and unit e'.

Using the stability methods of linear functional equations, we prove that every almost linear mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism when $h(2^n u \circ y) = h(2^n u) \circ h(y)$, $h(3^n u \circ y) = h(3^n u) \circ h(y)$ or $h(q^n u \circ y) = h(q^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, and that every almost linear almost multiplicative mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism when $h(2x) = 2h(x)$, $h(3x) = 3h(x)$ or $h(qx) = qh(x)$ for all $x \in \mathcal{A}$. We moreover prove the Cauchy–Rassias stability of Poisson JC^*-algebra homomorphisms in Poisson JC^*-algebras.

2 Homomorphisms between Poisson JC^*-algebras

Definition 2.1. A $\mathbb{C}$-linear mapping $H : \mathcal{A} \to \mathcal{B}$ is called a *Poisson JC^*-algebra homomorphism* if $H : \mathcal{A} \to \mathcal{B}$ satisfies

$$H(x \circ y) = H(x) \circ H(y),$$
$$H(\{x, y\}) = \{H(x), H(y)\}$$

for all $x, y \in \mathcal{A}$.

We are going to investigate Poisson JC^*-algebra homomorphisms between Poisson JC^*-algebras associated with the Cauchy functional equation.

Theorem 2.1. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(2^n u \circ y) = h(2^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exists a function $\varphi : \mathcal{A}^4 \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z, w) := \sum_{j=0}^{\infty} 2^{-j} \varphi(2^j x, 2^j y, 2^j z, 2^j w) < \infty, \tag{2.i}$$

$$\|h(\mu x + \mu y + \{z, w\}) - \mu h(x) - \mu h(y) - \{h(z), h(w)\}\| \leq \varphi(x, y, z, w) \tag{2.ii}$$

for all $\mu \in \mathbb{T}^1 := \{\lambda \in \mathbb{C} \mid |\lambda| = 1\}$, and all $x, y, z, w \in \mathcal{A}$. Assume that (2.iii) $\lim_{n\to\infty} \frac{h(2^n e)}{2^n} = e'$. *Then the mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.*

Proof. Put $z = w = 0$ and $\mu = 1 \in \mathbb{T}^1$ in (2.ii). It follows from Găvruta's Theorem [2] that there exists a unique additive mapping $H : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \leq \frac{1}{2}\widetilde{\varphi}(x, x, 0, 0) \tag{2.iv}$$

for all $x \in \mathcal{A}$. The additive mapping $H : \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{2^n} h(2^n x) \tag{2.1}$$

for all $x \in \mathcal{A}$.

By the assumption, for each $\mu \in \mathbb{T}^1$,

$$\|h(2^n \mu x) - 2\mu h(2^{n-1}x)\| \le \varphi(2^{n-1}x, 2^{n-1}x, 0, 0)$$

for all $x \in \mathcal{A}$. And one can show that

$$\begin{aligned}\|\mu h(2^n x) - 2\mu h(2^{n-1}x)\| &\le |\mu| \cdot \|h(2^n x) - 2h(2^{n-1}x)\| \\ &\le \varphi(2^{n-1}x, 2^{n-1}x, 0, 0)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$. So

$$\begin{aligned}\|h(2^n \mu x) - \mu h(2^n x)\| \le &\|h(2^n \mu x) - 2\mu h(2^{n-1}x)\| \\ &+ \|2\mu h(2^{n-1}x) - \mu h(2^n x)\| \\ \le &\varphi(2^{n-1}x, 2^{n-1}x, 0, 0) + \varphi(2^{n-1}x, 2^{n-1}x, 0, 0)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$. Thus $2^{-n}\|h(2^n \mu x) - \mu h(2^n x)\| \to 0$ as $n \to \infty$ for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$. Hence

$$H(\mu x) = \lim_{n\to\infty} \frac{h(2^n \mu x)}{2^n} = \lim_{n\to\infty} \frac{\mu h(2^n x)}{2^n} = \mu H(x) \tag{2.2}$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$.

Now let $\lambda \in \mathbb{C}$ ($\lambda \neq 0$) and M an integer greater than $4|\lambda|$. Then $|\frac{\lambda}{M}| < \frac{1}{4} < 1 - \frac{2}{3} = \frac{1}{3}$. By [5, Theorem 1], there exist three elements $\mu_1, \mu_2, \mu_3 \in \mathbb{T}^1$ such that $3\frac{\lambda}{M} = \mu_1 + \mu_2 + \mu_3$. And $H(x) = H(3 \cdot \frac{1}{3}x) = 3H(\frac{1}{3}x)$ for all $x \in \mathcal{A}$. So $H(\frac{1}{3}x) = \frac{1}{3}H(x)$ for all $x \in \mathcal{A}$. Thus by (2.2)

$$\begin{aligned}H(\lambda x) &= H\left(\frac{M}{3} \cdot 3\frac{\lambda}{M}x\right) = M \cdot H\left(\frac{1}{3} \cdot 3\frac{\lambda}{M}x\right) = \frac{M}{3}H\left(3\frac{\lambda}{M}x\right) \\ &= \frac{M}{3}H(\mu_1 x + \mu_2 x + \mu_3 x) = \frac{M}{3}\big(H(\mu_1 x) + H(\mu_2 x) + H(\mu_3 x)\big) \\ &= \frac{M}{3}(\mu_1 + \mu_2 + \mu_3)H(x) = \frac{M}{3} \cdot 3\frac{\lambda}{M}H(x) \\ &= \lambda H(x)\end{aligned}$$

for all $x \in \mathcal{A}$. Hence

$$H(\zeta x + \eta y) = H(\zeta x) + H(\eta y) = \zeta H(x) + \eta H(y)$$

for all $\zeta,\ \eta \in \mathbb{C}(\zeta, \eta \neq 0)$ and all $x, y \in \mathcal{A}$. And $H(0x) = 0 = 0H(x)$ for all $x \in \mathcal{A}$. So the unique additive mapping $H : \mathcal{A} \to \mathcal{B}$ is a $\mathbb{C}$-linear mapping.

Since $h(2^n u \circ y) = h(2^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$,

$$\begin{aligned} H(u \circ y) &= \lim_{n\to\infty} \frac{1}{2^n} h(2^n u \circ y) \\ &= \lim_{n\to\infty} \frac{1}{2^n} h(2^n u) \circ h(y) = H(u) \circ h(y) \end{aligned} \tag{2.3}$$

for all $y \in \mathcal{A}$ and all $u \in \mathcal{U}(\mathcal{A})$. By the additivity of H and (2.3),

$$2^n H(u \circ y) = H(2^n u \circ y) = H(u \circ (2^n y)) = H(u) \circ h(2^n y)$$

for all $y \in \mathcal{A}$ and all $u \in \mathcal{U}(\mathcal{A})$. Hence

$$H(u \circ y) = \frac{1}{2^n} H(u) \circ h(2^n y) = H(u) \circ \frac{1}{2^n} h(2^n y) \tag{2.4}$$

for all $y \in \mathcal{A}$ and all $u \in \mathcal{U}(\mathcal{A})$. Taking the limit in (2.4) as $n \to \infty$, we obtain

$$H(u \circ y) = H(u) \circ H(y) \tag{2.5}$$

for all $y \in \mathcal{A}$ and all $u \in \mathcal{U}(\mathcal{A})$. Since H is $\mathbb{C}$-linear and each $x \in \mathcal{A}$ is a finite linear combination of unitary elements (see [6, Theorem 4.1.7]), i.e., $x = \sum_{j=1}^m \lambda_j u_j$ $(\lambda_j \in \mathbb{C}, u_j \in \mathcal{U}(\mathcal{A}))$,

$$\begin{aligned} H(x \circ y) &= H\Big(\sum_{j=1}^m \lambda_j u_j \circ y\Big) = \sum_{j=1}^m \lambda_j H(u_j \circ y) = \sum_{j=1}^m \lambda_j H(u_j) \circ H(y) \\ &= H\Big(\sum_{j=1}^m \lambda_j u_j\Big) \circ H(y) = H(x) \circ H(y) \end{aligned}$$

for all $x, y \in \mathcal{A}$.

By (2.iii), (2.3) and (2.5),

$$H(y) = H(e \circ y) = H(e) \circ h(y) = e' \circ h(y) = h(y)$$

for all $y \in \mathcal{A}$. So

$$H(y) = h(y)$$

for all $y \in \mathcal{A}$.

It follows from (2.1) that

$$H(x) = \lim_{n\to\infty} \frac{h(2^{2n}x)}{2^{2n}} \tag{2.6}$$

for all $x \in \mathcal{A}$. Let $x = y = 0$ in (2.ii). Then we get

$$\|h(\{z, w\}) - \{h(z), h(w)\}\| \le \varphi(0, 0, z, w)$$

for all $z, w \in \mathcal{A}$. So

$$\begin{aligned}\frac{1}{2^{2n}}\|h(\{2^n z, 2^n w\}) - \{h(2^n z), h(2^n w)\}\| &\le \frac{1}{2^{2n}}\varphi(0, 0, 2^n z, 2^n w)\\ &\le \frac{1}{2^{n}}\varphi(0, 0, 2^n z, 2^n w)\end{aligned} \tag{2.7}$$

for all $z, w \in \mathcal{A}$. By (2.i), (2.6), and (2.7),

$$\begin{aligned}H(\{z, w\}) &= \lim_{n\to\infty} \frac{h(2^{2n}\{z, w\})}{2^{2n}} = \lim_{n\to\infty} \frac{h(\{2^n z, 2^n w\})}{2^{2n}}\\ &= \lim_{n\to\infty} \frac{1}{2^{2n}}\left\{h(2^n z), h(2^n w)\right\} = \lim_{n\to\infty}\left\{\frac{h(2^n z)}{2^n}, \frac{h(2^n w)}{2^n}\right\}\\ &= \{H(z), H(w)\}\end{aligned}$$

for all $z, w \in \mathcal{A}$.

Therefore, the mapping $h\colon \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism, as desired. □

Corollary 2.2. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(2^n u \circ y) = h(2^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exist constants $\theta \ge 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned}\|h(\mu x + \mu y + \{z, w\}) - &\mu h(x) - \mu h(y) - \{h(z), h(w)\}\|\\ &\le \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$, and all $x, y, z, w \in \mathcal{A}$. Assume that $\lim_{n\to\infty} \frac{h(2^n e)}{2^n} = e'$. Then the mapping $h\colon \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.

Proof. Define $\varphi(x, y, z, w) = \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p)$, and apply Theorem 2.1. □

Theorem 2.3. *Let $h: \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(2^n u \circ y) = h(2^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exists a function $\varphi: \mathcal{A}^4 \to [0, \infty)$ satisfying* (2.i) *and* (2.iii) *such that*

$$\begin{aligned}\|h(\mu x + \mu y + \{z, w\}) - \mu h(x) - \mu h(y) - \{h(z), h(w)\}\| \\ \leq \varphi(x, y, z, w)\end{aligned} \tag{2.v}$$

for $\mu = 1, i$, and all $x, y, z, w \in \mathcal{A}$. If $h(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in \mathcal{A}$, then the mapping $h: \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.

Proof. Put $z = w = 0$ and $\mu = 1$ in (2.v). By the same reasoning as in the proof of Theorem 2.1, there exists a unique additive mapping $H: \mathcal{A} \to \mathcal{B}$ satisfying (2.iv). The additive mapping $H: \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{2^n} h(2^n x)$$

for all $x \in \mathcal{A}$. By the same reasoning as in the proof of [12, Theorem], the additive mapping $H: \mathcal{A} \to \mathcal{B}$ is $\mathbb{R}$-linear.

Put $y = z = w = 0$ and $\mu = i$ in (2.v). By the same method as in the proof of Theorem 2.1, one can obtain that

$$H(ix) = \lim_{n\to\infty} \frac{h(2^n ix)}{2^n} = \lim_{n\to\infty} \frac{ih(2^n x)}{2^n} = iH(x)$$

for all $x \in \mathcal{A}$. For each element $\lambda \in \mathbb{C}$, $\lambda = s + it$, where $s, t \in \mathbb{R}$. So

$$\begin{aligned}H(\lambda x) &= H(sx + itx) = sH(x) + tH(ix) = sH(x) + itH(x) \\ &= (s + it)H(x) = \lambda H(x)\end{aligned}$$

for all $\lambda \in \mathbb{C}$ and all $x \in \mathcal{A}$. So

$$H(\zeta x + \eta y) = H(\zeta x) + H(\eta y) = \zeta H(x) + \eta H(y)$$

for all $\zeta, \eta \in \mathbb{C}$, and all $x, y \in \mathcal{A}$. Hence the additive mapping $H: \mathcal{A} \to \mathcal{B}$ is $\mathbb{C}$-linear.

The rest of the proof is the same as in the proof of Theorem 2.1. □

Theorem 2.4. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(2x) = 2h(x)$ for all $x \in \mathcal{A}$ for which there exists a function $\varphi\colon \mathcal{A}^4 \to [0, \infty)$ satisfying* (2.i), (2.ii) *and* (2.iii) *such that*

$$\|h(2^n u \circ y) - h(2^n u) \circ h(y)\| \leq \varphi(u, y, 0, 0) \tag{2.vi}$$

for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \ldots$. Then the mapping $h\colon \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.

Proof. By the same reasoning as in the proof of Theorem 2.1, there exists a unique $\mathbb{C}$-linear mapping $H\colon \mathcal{A} \to \mathcal{B}$ satisfying (2.iv).

By (2.vi) and the assumption that $h(2x) = 2h(x)$ for all $x \in \mathcal{A}$,

$$\begin{aligned}\|h(2^n u \circ y) - h(2^n u) \circ h(y)\| &= \frac{1}{4^m}\|h(2^m 2^n u \circ 2^m y) - h(2^m 2^n u) \circ h(2^m y)\| \\ &\leq \frac{1}{4^m}\varphi(2^m u, 2^m y, 0, 0) \\ &\leq \frac{1}{2^m}\varphi(2^m u, 2^m y, 0, 0),\end{aligned}$$

which tends to zero as $m \to \infty$ by (2.i). So

$$h(2^n u \circ y) = h(2^n u) \circ h(y)$$

for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$. But by (2.1),

$$H(x) = \lim_{n\to\infty} \frac{1}{2^n} h(2^n x) = h(x)$$

for all $x \in \mathcal{A}$.

The rest of the proof is the same as in the proof of Theorem 2.1. $\square$

Now we are going to investigate Poisson JC^*-algebra homomorphisms between Poisson JC^*-algebras associated with the Jensen functional equation.

Theorem 2.5. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(3^n u \circ y) = h(3^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exists a function $\varphi\colon (\mathcal{A} \setminus \{0\})^4 \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z, w) := \sum_{j=0}^{\infty} 3^{-j}\varphi(3^j x, 3^j y, 3^j z, 3^j w) < \infty, \tag{2.vii}$$

$$\begin{aligned}&\|2h\Big(\frac{\mu x + \mu y + \{z, w\}}{2}\Big) - \mu h(x) - \mu h(y) - \{h(z), h(w)\}\| \\ &\qquad\qquad \leq \varphi(x, y, z, w)\end{aligned} \tag{2.viii}$$

for all $\mu \in \mathbb{T}^1$, and all $x, y, z, w \in \mathcal{A}$. Assume that $\lim_{n\to\infty} \frac{h(3^n e)}{3^n} = e'$. *Then the mapping $h\colon \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.*

Proof. Put $z = w = 0$ and $\mu = 1 \in \mathbb{T}^1$ in (2.viii). It follows from Jun and Lee's Theorem [4, Theorem 1] that there exists a unique additive mapping $H\colon \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \le \frac{1}{3}(\widetilde{\varphi}(x, -x, 0, 0) + \widetilde{\varphi}(-x, 3x, 0, 0))$$

for all $x \in \mathcal{A} \setminus \{0\}$. The additive mapping $H\colon \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{3^n} h(3^n x)$$

for all $x \in \mathcal{A}$.

By the assumption, for each $\mu \in \mathbb{T}^1$,

$$\|2h(3^n \mu x) - \mu h(2 \cdot 3^{n-1}x) - \mu h(4 \cdot 3^{n-1}x)\| \le \varphi(2 \cdot 3^{n-1}x, 4 \cdot 3^{n-1}x, 0, 0)$$

for all $x \in \mathcal{A} \setminus \{0\}$. And one can show that

$$\begin{aligned}\|\mu h(2 \cdot 3^{n-1}x) &+ \mu h(4 \cdot 3^{n-1}x) - 2\mu h(3^n x)\| \\ &\le |\mu| \cdot \|h(2 \cdot 3^{n-1}x) + h(4 \cdot 3^{n-1}x) - 2h(3^n x)\| \\ &\le \varphi(2 \cdot 3^{n-1}x, 4 \cdot 3^{n-1}x, 0, 0)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A} \setminus \{0\}$. So

$$\begin{aligned}\|h(3^n \mu x) - \mu h(3^n x)\| =& \|h(3^n \mu x) - \frac{1}{2}\mu h(2 \cdot 3^{n-1}x) - \frac{1}{2}\mu h(4 \cdot 3^{n-1}x) \\ &+ \frac{1}{2}\mu h(2 \cdot 3^{n-1}x) + \frac{1}{2}\mu h(4 \cdot 3^{n-1}x) - \mu h(3^n x)\| \\ \le& \frac{1}{2}\|2h(3^n \mu x) - \mu h(2 \cdot 3^{n-1}x) - \mu h(4 \cdot 3^{n-1}x)\| \\ &+ \frac{1}{2}\|\mu h(2 \cdot 3^{n-1}x) + \mu h(4 \cdot 3^{n-1}x) - 2\mu h(3^n x)\| \\ \le& \frac{2}{2}\varphi(2 \cdot 3^{n-1}x, 4 \cdot 3^{n-1}x, 0, 0)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A} \setminus \{0\}$. Thus $3^{-n}\|h(3^n \mu x) - \mu h(3^n x)\| \to 0$ as $n \to \infty$ for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A} \setminus \{0\}$. Hence

$$H(\mu x) = \lim_{n\to\infty} \frac{h(3^n \mu x)}{3^n} = \lim_{n\to\infty} \frac{\mu h(3^n x)}{3^n} = \mu H(x)$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A} \setminus \{0\}$.

By the same reasoning as in the proof of Theorem 2.1, the unique additive mapping $H : \mathcal{A} \to \mathcal{B}$ is a $\mathbb{C}$-linear mapping.

By a similar method to the proof of Theorem 2.1, one can show that the mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism. $\square$

Corollary 2.6. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(3^n u \circ y) = h(3^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned}\|2h\Big(\frac{\mu x + \mu y + \{z, w\}}{2}\Big) &- \mu h(x) - \mu h(y) - \{h(z), h(w)\}\| \\ &\leq \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$, and all $x, y, z, w \in \mathcal{A} \setminus \{0\}$. Assume $\lim_{n\to\infty} \frac{h(3^n e)}{3^n} = e'$. Then the mapping $h : \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.

Proof. Define $\varphi(x, y, z, w) = \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p)$, and apply Theorem 2.5. $\square$

One can obtain similar results to Theorems 2.3 and 2.4 for the Jensen functional equation.

Finally, we are going to investigate Poisson JC^*-algebra homomorphisms between Poisson JC^*-algebras associated with the Trif functional equation.

Theorem 2.7. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(q^n u \circ y) = h(q^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exists a function $\varphi : \mathcal{A}^{d+2} \to [0, \infty)$ such that*

$$\widetilde{\varphi}\big(x_1, \cdots, x_d, z, w\big) := \sum_{j=0}^{\infty} q^{-j}\varphi\big(q^j x_1, \cdots, q^j x_d, q^j z, q^j w\big) < \infty, \quad \text{(2.ix)}$$

$$\begin{aligned}&\|d\ {}_{d-2}C_{l-2} h\left(\frac{\mu x_1 + \cdots + \mu x_d}{d} + \frac{\{z, w\}}{d\ {}_{d-2}C_{l-2}}\right) + {}_{d-2}C_{l-1}\sum_{j=1}^{d} \mu h(x_j) \\ &- l \sum_{1\leq j_1<\cdots<j_l\leq d} \mu h\left(\frac{x_{j_1} + \cdots + x_{j_l}}{l}\right) - \{h(z), h(w)\}\| \qquad \text{(2.x)} \\ &\leq \varphi(x_1, \cdots, x_d, z, w)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$, and all $x_1, \cdots, x_d, z, w \in \mathcal{A}$. Assume $\lim_{n\to\infty} \frac{h(q^n e)}{q^n} = e'$. Then the mapping $h: \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.

Proof. Put $z = w = 0$ and $\mu = 1 \in \mathbb{T}^1$ in (2.x). It follows from Trif's Theorem [18, Theorem 3.1] that there exists a unique additive mapping $H: \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \le \frac{1}{l \cdot {}_{d-1}C_{l-1}} \widetilde{\varphi}(qx, \underbrace{rx, \cdots, rx}_{d-1 \text{ times}}, 0, 0)$$

for all $x \in \mathcal{A}$. The additive mapping $H: \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{q^n} h(q^n x)$$

for all $x \in \mathcal{A}$.

Put $x_1 = \cdots = x_d = x$ and $z = w = 0$ in (2.x). For each $\mu \in \mathbb{T}^1$,

$$\|d\ {}_{d-2}C_{l-2}(h(\mu x) - \mu h(x))\| \le \varphi(\underbrace{x, \cdots, x}_{d \text{ times}}, 0, 0)$$

for all $x \in \mathcal{A}$. So

$$q^{-n}\|d\ {}_{d-2}C_{l-2}(h(\mu q^n x) - \mu h(q^n x))\| \le q^{-n}\varphi(\underbrace{q^n x, \cdots, q^n x}_{d \text{ times}}, 0, 0)$$

for all $x \in \mathcal{A}$. By (2.ix),

$$q^{-n}\|d\ {}_{d-2}C_{l-2}(h(\mu q^n x) - \mu h(q^n x))\| \to 0$$

as $n \to \infty$ for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$. Thus

$$q^{-n}\|h(\mu q^n x) - \mu h(q^n x)\| \to 0$$

as $n \to \infty$ for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$. Hence

$$H(\mu x) = \lim_{n\to\infty} \frac{h(q^n \mu x)}{q^n} = \lim_{n\to\infty} \frac{\mu h(q^n x)}{q^n} = \mu H(x)$$

for all $\mu \in \mathbb{T}^1$ and all $x \in \mathcal{A}$.

By the same reasoning as in the proof of Theorem 2.1, the unique additive mapping $H: \mathcal{A} \to \mathcal{B}$ is a $\mathbb{C}$-linear mapping.

By a similar method to the proof of Theorem 2.1, one can show that the mapping $h: \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism. □

Corollary 2.8. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping satisfying $h(0) = 0$ and $h(q^n u \circ y) = h(q^n u) \circ h(y)$ for all $y \in \mathcal{A}$, all $u \in \mathcal{U}(\mathcal{A})$ and $n = 0, 1, 2, \cdots$, for which there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that*

$$\|d\ {}_{d-2}C_{l-2} h\Big(\frac{\mu x_1 + \cdots + \mu x_d}{d} + \frac{\{z, w\}}{d\ {}_{d-2}C_{l-2}}\Big) + {}_{d-2}C_{l-1} \sum_{j=1}^{d} \mu h(x_j)$$
$$- l \sum_{1 \leq j_1 < \cdots < j_l \leq d} \mu h\Big(\frac{x_{j_1} + \cdots + x_{j_l}}{l}\Big) - \{h(z), h(w)\}\|$$
$$\leq \theta\big(\sum_{j=1}^{d} ||x_j||^p + ||z||^p + ||w||^p\big)$$

for all $\mu \in \mathbb{T}^1$, and all $x_1, \cdots, x_d, z, w \in \mathcal{A}$. Assume $\lim_{n\to\infty} \frac{h(q^n e)}{q^n} = e'$. Then the mapping $h\colon \mathcal{A} \to \mathcal{B}$ is a Poisson JC^*-algebra homomorphism.

Proof. Define $\varphi(x_1, \cdots, x_d, z, w) = \theta(\sum_{j=1}^{d} ||x_j||^p + ||z||^p + ||w||^p)$, and apply Theorem 2.7. □

One can obtain similar results to Theorems 2.3 and 2.4 for the Trif functional equation.

3 Stability of homomorphisms in Poisson JC^*-algebras

We are going to show the Cauchy–Rassias stability of homomorphisms in Poisson JC^*-algebras associated with the Cauchy functional equation.

Theorem 3.1. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exists a function $\varphi\colon \mathcal{A}^6 \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z, w, a, b) := \sum_{j=0}^{\infty} 2^{-j} \varphi(2^j x, 2^j y, 2^j z, 2^j w, 2^j a, 2^j b) < \infty, \tag{3.i}$$

$$\|h(\mu x + \mu y + \{z, w\} + a \circ b) - \mu h(x) - \mu h(y) - \{h(z), h(w)\}$$
$$- h(a) \circ h(b)\| \leq \varphi(x, y, z, w, a, b) \tag{3.ii}$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z, w, a, b \in \mathcal{A}$. Then there exists a unique Poisson JC^*-algebra homomorphism $H\colon \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \leq \frac{1}{2} \widetilde{\varphi}(x, x, 0, 0, 0, 0) \tag{3.iii}$$

for all $x \in \mathcal{A}$.

Proof. Put $z = w = a = b = 0$ and $\mu = 1 \in \mathbb{T}^1$ in (3.ii). It follows from Găvruta's Theorem [2] that there exists a unique additive mapping $H: \mathcal{A} \to \mathcal{B}$ satisfying (3.iii). The additive mapping $H: \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{2^n} h(2^n x)$$

for all $x \in \mathcal{A}$.

The rest of the proof is similar to the proof of Theorem 2.1. □

Corollary 3.2. *Let $h: \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned} \|h(\mu x + \mu y + \{z, w\} + a \circ b) - \mu h(x) - \mu h(y) \\ - \{h(z), h(w)\} - h(a) \circ h(b)\| \\ \leq \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p + ||a||^p + ||b||^p) \end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z, w, a, b \in \mathcal{A}$. Then there exists a unique Poisson JC^*-algebra homomorphism $H: \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \leq \frac{2\theta}{2 - 2^p} ||x||^p$$

for all $x \in \mathcal{A}$.

Proof. Define

$$\varphi(x, y, z, w, a, b) = \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p + ||a||^p + ||b||^p),$$

and apply Theorem 3.1. □

Theorem 3.3. *Let $h: \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exists a function $\varphi: \mathcal{A}^6 \to [0, \infty)$ satisfying* (3.i) *such that*

$$\begin{aligned} \|h(\mu x + \mu y + \{z, w\} + a \circ b) - \mu h(x) - \mu h(y) \\ - \{h(z), h(w)\} - h(a) \circ h(b)\| \\ \leq \varphi(x, y, z, w, a, b) \end{aligned}$$

for $\mu = 1, i$, and all $x, y, z, w, a, b \in \mathcal{A}$. If $h(tx)$ is continuous in $t \in \mathbb{R}$ for each fixed $x \in \mathcal{A}$, then there exists a unique Poisson JC^*-algebra homomorphism $H: \mathcal{A} \to \mathcal{B}$ satisfying (3.iii).

Proof. The proof is similar to the proof of Theorem 2.3. $\square$

We are going to show the Cauchy–Rassias stability of homomorphisms in Poisson JC^*-algebras associated with the Jensen functional equation.

Theorem 3.4. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exists a function $\varphi\colon (A \setminus \{0\})^6 \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x, y, z, w, a, b) = \sum_{j=0}^{\infty} 3^{-j}\varphi(3^j x, 3^j y, 3^j z, 3^j w, 3^j a, 3^j b) < \infty,$$

$$\begin{aligned} \|2h\left(\frac{\mu x + \mu y + \{z, w\} + a \circ b}{2}\right) - \mu h(x) - \mu h(y) \\ -\{h(z), h(w)\} - h(a) \circ h(b)\| \le \varphi(x, y, z, w, a, b) \end{aligned} \tag{3.iv}$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z, w, a, b \in \mathcal{A} \setminus \{0\}$. Then there exists a unique Poisson JC^*-algebra homomorphism $H\colon \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \le \frac{1}{3}\big(\widetilde{\varphi}(x, -x, 0, 0, 0, 0) + \widetilde{\varphi}(-x, 3x, 0, 0, 0, 0)\big) \tag{3.v}$$

for all $x \in \mathcal{A} \setminus \{0\}$.

Proof. Put $z = w = a = b = 0$ and $\mu = 1 \in \mathbb{T}^1$ in (3.iv). It follows from Jun and Lee's Theorem [4, Theorem 1] that there exists a unique additive mapping $H\colon \mathcal{A} \to \mathcal{B}$ satisfying (3.v). The additive mapping $H\colon \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{3^n} h(3^n x)$$

for all $x \in \mathcal{A}$.

The rest of the proof is similar to the proof of Theorem 2.5. $\square$

Corollary 3.5. *Let $h\colon \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exist constants $\theta \ge 0$ and $p \in [0, 1)$ such that*

$$\|2h\left(\frac{\mu x + \mu y + \{z, w\} + a \circ b}{2}\right) - \mu h(x) - \mu h(y)$$

$$-\{h(z), h(w)\} - h(a) \circ h(b)\|$$

$$\le \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p + ||a||^p + ||b||^p)$$

for all $\mu \in \mathbb{T}^1$ and all $x, y, z, w, a, b \in \mathcal{A} \setminus \{0\}$. Then there exists a unique Poisson JC^*-algebra homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \leq \frac{3 + 3^p}{3 - 3^p} \theta ||x||^p$$

for all $x \in \mathcal{A} \setminus \{0\}$.

Proof. Define

$$\varphi(x, y, z, w, a, b) = \theta(||x||^p + ||y||^p + ||z||^p + ||w||^p + ||a||^p + ||b||^p),$$

and apply Theorem 3.4. □

One can obtain a similar result to Theorem 3.3 for the Jensen functional equation.

Now we are going to show the Cauchy–Rassias stability of homomorphisms in Poisson JC^*-algebras associated with the Trif functional equation.

Theorem 3.6. *Let $h : \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exists a function $\varphi : \mathcal{A}^{d+4} \to [0, \infty)$ such that*

$$\widetilde{\varphi}(x_1, \cdots, x_d, z, w, a, b) := \sum_{j=0}^{\infty} q^{-j} \varphi(q^j x_1, \cdots, q^j x_d, q^j z, q^j w, q^j a, q^j b) < \infty,$$

$$\begin{aligned} &\|d\ {}_{d-2}C_{l-2} h\left(\frac{\mu x_1 + \cdots + \mu x_d}{d} + \frac{\{z, w\} + a \circ b}{d\ {}_{d-2}C_{l-2}}\right) + {}_{d-2}C_{l-1} \sum_{j=1}^{d} \mu h(x_j) \\ &- l \sum_{1 \leq j_1 < \cdots < j_l \leq d} \mu h\left(\frac{x_{j_1} + \cdots + x_{j_l}}{l}\right) - \{h(z), h(w)\} - h(a) \circ h(b)\| \\ &\leq \varphi(x_1, \cdots, x_d, z, w, a, b) \end{aligned} \tag{3.vi}$$

for all $\mu \in \mathbb{T}^1$ and all $x_1, \cdots, x_d, z, w, a, b \in \mathcal{A}$. Then there exists a unique Poisson JC^*-algebra homomorphism $H : \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \leq \frac{1}{l \cdot {}_{d-1}C_{l-1}} \widetilde{\varphi}(qx, \underbrace{rx, \cdots, rx}_{d-1 \text{ times}}, 0, 0, 0, 0) \tag{3.vii}$$

for all $x \in \mathcal{A}$.

Proof. Put $z = w = a = b = 0$ and $\mu = 1 \in \mathbb{T}^1$ in (3.vi). It follows from Trif's Theorem [18, Theorem 3.1] that there exists a unique additive mapping $H: \mathcal{A} \to \mathcal{B}$ satisfying (3.vii). The additive mapping $H: \mathcal{A} \to \mathcal{B}$ is given by

$$H(x) = \lim_{n\to\infty} \frac{1}{q^n} h(q^n x)$$

for all $x \in \mathcal{A}$.

The rest of the proof is similar to the proof of Theorem 2.7. □

Corollary 3.7. *Let $h: \mathcal{A} \to \mathcal{B}$ be a mapping with $h(0) = 0$ for which there exist constants $\theta \geq 0$ and $p \in [0, 1)$ such that*

$$\begin{aligned}&\| d\ {}_{d-2}C_{l-2} h\left(\frac{\mu x_1 + \cdots + \mu x_d}{d} + \frac{\{z, w\} + a \circ b}{d\ {}_{d-2}C_{l-2}}\right) + {}_{d-2}C_{l-1} \sum_{j=1}^{d} \mu h(x_j) \\ &- l \sum_{1 \leq j_1 < \cdots < j_l \leq d} \mu h\left(\frac{x_{j_1} + \cdots + x_{j_l}}{l}\right) - \{h(z), h(w)\} - h(a) \circ h(b) \| \\ &\leq \theta\left(\sum_{j=1}^{d} ||x_j||^p + ||z||^p + ||w||^p + ||a||^p + ||b||^p\right)\end{aligned}$$

for all $\mu \in \mathbb{T}^1$ and all $x_1, \cdots, x_d, z, w, a, b \in \mathcal{A}$. Then there exists a unique Poisson JC^*-algebra homomorphism $H: \mathcal{A} \to \mathcal{B}$ such that

$$\|h(x) - H(x)\| \leq \frac{q^{1-p}(q^p + (d-1)r^p)\theta}{l\ {}_{d-1}C_{l-1}(q^{1-p} - 1)} ||x||^p$$

for all $x \in \mathcal{A}$.

Proof. Define

$$\varphi(x_1, \cdots, x_d, z, w, a, b) = \theta\left(\sum_{j=1}^{d} ||x_j||^p + ||z||^p + ||w||^p + ||a||^p + ||b||^p\right),$$

and apply Theorem 3.6. □

One can obtain a similar result to Theorem 3.3 for the Trif functional equation.

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